Proceedings

14th UFPR Symposium on Partial Differential Equations

Graduate Program in Mathematics

Federal University of Paraná

Proceedings of the

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Abstracts: Wednesday, April 7

UFPR - historic building at Santos Andrade Square
Photo: Marcos Solivan

Asymptotic behaviour of the energy to the viscoelastic wave equation with localized memory and supercritical nonlinearities

Valéria Neves Domingos Cavalcanti

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Abstract

We are concerned with the well-posedness of solutions as well as the decay rates of the energy related to the viscoelastic wave equation with localized memory with past history and supercritical source and damping terms, posed on a bounded domain in the three-dimensional euclidean space. This a joint work with M. Cavalcanti, T. D. Marchiori and C. M. Webler.

References

[1] Cavalcanti, Marcelo M.;Domingos Cavalcanti, Valéria N.;Jorge Silva, Márcio A. and Souza Franco, Alisson Y. Exponential stability for the wave model with localized memory in a past history framework, *J. Differential Equations*, 264, p. 6535–6584, 2018.

[2] Dafermos, Constantine M. Asymptotic stability in viscoelasticity, *Arch. Rational Mech. Anal.*, 37, p. 297– 308, 1970.

[3] Guo, Yanqiu;Rammaha, Mohammad A.;Sakuntasathien, Sawanya;Titi, Edriss S. and Toundykov, Daniel Hadamard well-posedness for a hyperbolic equation of viscoelasticity with supercritical sources and damping, *J. Differential Equations*, 257, p. 3778–3812, 2014.

The Vázquez maximum principle and the Landis conjecture for elliptic PDE

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Abstract

We develop a new, unified approach to the following two classical questions on elliptic PDE:

- the strong maximum principle for equations with non-Lipschitz nonlinearities,

- the at most exponential decay of solutions in the whole space or exterior domains.

Our results apply to divergence and nondivergence operators with locally unbounded lower-order coefficients, in a number of situations where all previous results required bounded ingredients. Our approach, which allows for relatively simple and short proofs, is based on a (weak) Harnack inequality with optimal dependence of the constants in the lower-order terms of the equation and the size of the domain, which we establish.

References

[1]   Sirakov, B. & Souplet, P.. The Vázquez maximum principle and the Landis conjecture for elliptic PDE with unbounded coefficients. *https://arxiv.org/abs/2010.08511*

New perspectives of uniform stability for thermo-viscoelastic Timoshenko beams

Marcio A. Jorge da Silva

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Abstract

In this talk we address some classical and non-classical Timoshenko systems under thermo-viscoelastic damping effects. The main goal is to investigate different and new thermoelastic Timoshenko systems with or without history, and with Fourier or Cattaneo law for heat conduction, by presenting how couplings affect the structure of the uniform decay of solutions when time goes to infinity.

The sensitivity of the Timoshenko system with respect to heat conduction laws and history terms is illustrated not only mathematically but also physically. To this purpose, we present some recent stability results obtained in [4], by regarding the physical modeling provided in in [1, 2] which in turn are based on the classical works [3, 5, 6, 7].

References

[1] Alves, M. O.; Caixeta, H. A.; Jorge Silva, M. A.; Rodrigues,J. H.; Almeida Júnior D. S. On a Timoshenko system with thermal coupling on both the bending moment and the shear force. *J. Evol. Equ.*, 20 (1), p. 295–320, 2020.

[2] Alves, M. O.; Gomes Tavares, E. H.; Jorge Silva, M. A.; Rodrigues, J. H. On Modeling and Uniform Stability of a Partially Dissipative Viscoelastic Timoshenko System, *SIAM Journal on Mathematical Analysis*, 51 (6), p. 4520– 4543, 2019.

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[4] Jorge Silva, M. A. & Reinhard, R. Effects of history and heat models on the stability of thermoelastic Timoshenko systems, *Journal of Differential Equations*, 275, p. 167–203, 2021.

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[6] Lagnese, J. E. & Lions, J. L. *Modelling, Analysis and Control of Thin Plates (Recherches en Mathématiques Appliquées Research Notes in App)* vol. 6. Paris: Mason, 1988.

[7] Prüss,J. *Evolutionary integral equations and applications.* Monographs in Mathematics, 87. Birkhäuser Basel, 1993.

Bound-waves due to sea and swell trigger the generation of freak-waves

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Abstract

In this talk we present a mechanism for the generation of freak-waves in the ocean. Such mechanism is based on an instability of narrow-banded homogeneous spectra to inhomogeneous disturbances. We show that when the sea state is in the presence of a swell, averaged bound-waves, that arise from their mutual interaction, drive the system away from the equilibrium. Then, by studying the non-linear evolution of the underlying sea state we find a significant increase in the probability of freak wave occurrence. This is joint work with prof. Michael Stiassnie.

References

[1]   Andrade, David & Stiassnie, Michael. Bound-waves due to sea and swell trigger the generation of freak-waves. *Journal of Ocean Engineering and Marine Energy* 6.4 , p. 399–414, 2020.

Large amplitude mode-2 internal solitary waves in three-layer flows

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Abstract

We consider a strongly nonlinear long wave model for large amplitude internal waves in a three-layer flow bounded above and below by rigid boundaries. The model extends the two-layer Miyata-Choi-Camassa (MCC) model (Miyata 1988; Choi & Camassa 1999) and is able to describe the propagation of long internal waves of both the first and second baroclinic modes. Solitary-wave solutions of the model are shown to be governed by a Hamiltonian system with two degrees of freedom. Emphasis is given to the solitary waves of the second baroclinic mode (mode-2) and their strongly nonlinear characteristics that fail to be captured by weakly nonlinear models. In addition to large amplitude mode-2 waves with single-hump profiles, new classes of mode-2 solutions, characterised by multi-humped wave profiles of large amplitude, are also found. The rationale behind the existence of such waves is explained based on the asymptotic limit when the density transition layer is thin. Our analytical predictions based on asymptotic theory are then corroborated by a numerical study of the full dynamical system.

References

[1]   Barros, R., Choi, W. and Milewski, P. A. Strongly nonlinear effects on internal solitary waves in three-layer flows, *J. Fluid Mech.* 883, 2020.

[2]   Choi, W. and Camassa, R. Fully nonlinear internal waves in a two-fluid system, *J. Fluid Mech.* 396, 1999.

[3]   Miyata, M. Long internal waves of large amplitude, In *Proceedings of the IUTAM Symposium on Nonlinear Water Waves* (ed. H. Horikawa & H. Maruo), pp. 399–406. Springer.

Abstracts: Wednesday, April 7

Botanical Garden of Curitiba
Photo: Daniel Castellano/SMCS

Sharp Weyl formula for operators on asymptotically Euclidean manifolds

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Abstract

We study the asymptotic behaviour of the eigenvalue counting function for self-adjoint, positive, elliptic linear operators, defined through classical weighted symbols of order $(1, 1)$, on an asymptotically Euclidean manifold X. We first prove a two term Weyl formula, improving previously known remainder estimates. Subsequently, we show that, under a geometric assumption on the Hamiltonian flow at infinity, there is a refined Weyl asymptotics with three terms. Finally, we illustrate the results by analysing the operator $\mathrm{op}^w(q)$ on $\mathbb{R}^d$, with $q(x, \xi) = (1 + |x|^2)(1 + |\xi|^2)$. This is joint work with Moritz Doll.

Estimates for a time-dependent multiplier with oscillatory and diffusive components

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University of São Paulo

Abstract

In this talk, we derive long time $L^p - L^q$ decay estimates, in the full range $1 \leq p \leq q \leq \infty$, for time-dependent multipliers in which an interplay between an oscillatory component and a diffusive component with different scaling appears. We estimate $\|m(t, \cdot)\|_{M_p^q}$ as $t \to \infty$ for multipliers of type

$$m(t, \xi) = e^{\pm i|\xi|^\sigma t - |\xi|^\theta t},$$

and suitable perturbations, under the assumption that the scaling of the diffusive component is worse, i.e., $\theta > \sigma$. These multipliers are, for instance, related to the fundamental solution to the Cauchy problem for the σ-evolution equation with structural damping:

$$u_{tt} + (-\Delta)^\sigma u + (-\Delta)^{\frac{\theta}{2}} u_t = 0, \quad t \geq 0, \ x \in \mathbb{R}^n,$$

in the so-called non-effective case $\sigma < \theta$.

This is a joint work with Marcello D'Abbicco (University of Bari).

Smoothing and Strichartz estimates for some time-degenerate Schrödinger operators

Serena Federico

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Ghent University

Abstract

In this talk I will analyze the validity of smoothing and Strichartz estimates for some classes of time-degenerate Schrödinger operators. In particular, we shall first investigate the local smoothing effect (both homogeneous and inhomogeneous) for time-degenerate Schrödinger operators of the form

$$\mathcal{L}_{\alpha,c} = i\partial_t + t^\alpha \Delta_x + c(t,x) \cdot \nabla_x, \quad \alpha > 0,$$

where $c(t,x)$ satisfies suitable conditions. As an application of the smoothing effect we will show the local well-posedness of the associated nonlinear Cauchy problem (see [1]).

Afterwards, we shall analyze Strichartz estimates for a class of operators similar to the previous one, that is of the form

$$\mathcal{L}_b := i\partial_t + b'(t)\Delta_x,$$

with b' satisfying suitable conditions. An application of these estimates will give a (different) local well-posedness result for a semilinear Cauchy problem associated with $\mathcal{L}_b$ (see [2]).

References

[1]   Serena, Federico and Gigliola, Staffilani,   Smoothing effect for time-degenerate Schrödinger operators. *Preprint. Arxiv https://arxiv.org/abs/2001.06708.* 2020.

[2]   Serena, Federico and Michael, Ruzhansky,   Smoothing and Strichartz estimates for degenerate Schrödinger-type equations.   *Preprint. Arxiv https://arxiv.org/abs/2005.01622.* 2020.

Diffusions and sources interacting in stationary models ruled by porous medium equations

Marcos da Silva Montenegro

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Abstract

This talk concerns existence and properties of positive solutions of the following problem in the presence of diffusion (a) and source (f):

$$\begin{cases} -\Delta(a(u)) &=& \lambda f(u) & \text{in } \Omega, \\ u &=& 0 & \text{on } \partial\Omega, \end{cases}$$

where Ω is a smooth bounded domain.

On the Asymptotic Stability for a Strongly Coupled Klein-Gordon System with Localized Damping

María Rosario Astudillo Rojas

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UFSC

Abstract

In this talk, we discuss the asymptotic behaviour of solutions of a strongly coupled Klein-Gordon system, posed in an inhomogeneous medium with smooth boundary, subject to a local damping distributed around a neighbourhood of the boundary, according to the Geometric Control Condition. We show that the energy of the system goes uniformly and exponentially to zero for all initial data of finite energy phase-space. The method of proof combines results by Burq and Gerard on microlocal analysis and L^p-L^q-Carleman estimates used to obtain a unique continuation property for coupled systems.

This is a joint work with M.M.Cavalcanti, V.N.Domingos Cavalcanti, S.Mansouri, V.H.Gonzalez Martinez and Z.Hajjej.

References

[1]   Astudillo, María; Moreira Cavalcanti, Marcelo; Correa, Wellington J.; Cavalcanti, Valéria Neves Domingos; Martinez, Victor Gonzalez; Vicente, André Asymptotic stability for a strongly coupled Klein-Gordon system in an inhomogeneous medium with locally distributed damping. *Journal of Differential Equations*, v. 268, p. 447-489, 2020.

Abstracts: Friday, April 9

Oscar Niemeyer Museum
Photo: Cesar Brustolin/SMCS

Periodic functions on isolated time scales

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Abstract

In this work, we formulate the definition of periodicity for isolated time scales. This provides the basis for future research regarding periodicity on time scales with a positive graininess. The introduced definition is consistent with the known formulation in the discrete and quantum calculus setting. Using the definition of periodicity, we discuss the existence and uniqueness of T–periodic solutions to a family of linear dynamic equations on isolated time scales. Examples in quantum calculus and mixed isolated time scales are presented throughout. This is a joint work with Martin Bohner and Sabrina Streipert.

The Cauchy problem for 3-evolution equations in Gevrey-type spaces

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Abstract

We consider the initial value problem for evolution operators of the form

$$P(t, x, D_t, D_x) = D_t u + a_p(t) D_x^p u(t,x) + \sum_{i=0}^{p-1} a_i(t,x) D_x^i u(t,x),$$

with $D = \frac{1}{i}\partial$, $p \geq 2$, $t \in [0,T], x \in \mathbb{R}$, $a_p \in C([0,T], \mathbb{R})$, $a_p(t) \neq 0$ for all $t \in [0,T]$ and $a_i(t,x) \in C([0,T], C^\infty(\mathbb{R}; \mathbb{C}))$ $i = 0, \ldots, p-1$. The operator P is known in literature as $p-$evolution operator. The condition that a_p is real valued means that the principal symbol of P (in the sense of Petrowski) has the real characteristic $\tau = -a_p(t)\xi^p$; by the Lax-Mizohata theorem, this condition is necessary to have a unique solution, in Sobolev spaces, of the Cauchy problem associated to P. When the coefficients $a_i(t,x)$, $i = 0, \ldots, p-1$ are real and uniformly bounded together with all their x-derivatives, it is well known that the Cauchy problem is well-posed in $L^2(\mathbb{R})$ and in Sobolev spaces H^m, $m \in \mathbb{R}$. If some of the coefficients $a_i(t,x)$ are complex valued, then some decay conditions at infinity on the imaginary part of the coefficients a_i are needed in order to obtain well-posedness either in $L^2(\mathbb{R})$, or in $H^\infty(\mathbb{R}) = \cap_{m\in\mathbb{R}} H^m(\mathbb{R})$, or in Gevrey classes, in general with a loss of derivatives. In particular, for Gevrey classes, there exist in the literature only results for the cases $p = 1, 2$, that is for strictly hyperbolic equations ($p = 1$) and for Schrödinger-type equations ($p = 2$). Here we focus on the case $p = 3$ and prove sufficient conditions on the coefficients a_i for the well posedness of the Cauchy problem with initial data in Gevrey-type spaces. To study this problem we need to introduce a suitable class of pseudo-differential operators of infinite order, that is with symbols admitting exponential growth at infinity and use a change of variable which can be expressed in terms of these operators. This approach is hopefully adaptable to study p-evolution equations also for $p > 3$. The results are obtained jointly with Alexandre Arias Junior (Federal University of Parana) and Alessia Ascanelli (University of Ferrara).

Gevrey solvability of a class of first order differential operators on the torus

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Abstract

In this talk we will deal with Gevrey global solvability on the N-dimensional torus ($\mathbb{T}^N \simeq \mathbb{R}^N/2\pi\mathbb{Z}^N$) to a class of nonlinear first order partial differential equations in the form $Lu - au - b\overline{u} = f$, where a, b, and f are Gevrey functions in $\mathbb{T}^N$ and L is a complex vector field defined on $\mathbb{T}^N$. Diophantine properties of the coefficients of L appear in a natural way in our results. Also, we present results in C^∞ context.

This is a joint work with Marcelo F. de Almeida (Federal University of Sergipe).

References

[1] Bergamasco, A. P. ; Dattori da Silva, P. L. ; Meziani, A. Solvability of a first order differential operator on the two-torus, *J. Math. Anal. Appl.*, 416, no. 1, p. 166–180, 2014.

[2] Almeida, M. F. de & Dattori da Silva, P. L. *Solvability of a class of first order differential operators on the torus*, Preprint.

Spectral analysis and applications for some control problems

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Abstract

In this talk, we will analyze the spectrum of some systems of partial differential equations that appear in beam dynamics (with and without thermal effects). Using this analysis and some Ingham-type theorems, we will solve some controllability problems.

A journey on issues dealing with path integration to describe wave functions

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Abstract

This talk gathers results from joint works with Felipe Federson, Everaldo Bonotto and Patrick Muldowney on Henstock path integration applied to Quantum Mechanics. In particular, we deal with equations involving Hamilton's principal function with Fresnel integrands.

References

[1]   Bonotto, E. M.; Federson, F.; Federson, M. The Schrodinger equation, path integration and applications, *Proceedings of the Singapore National Academy of Science*, v. 15 (1), p. 61–75, 2021.

[2]   Bonotto, E. M.; Federson, M.; Muldowney, P. The Black-Scholes equation with impulses at random times via generalized Riemann integral, *Proceedings of the Singapore National Academy of Science*, v. 15 (1), p. 45–59, 2021.

Auracaria angustifolia tree symbol of State of Paraná.