Sub-exponential decay and holomorphic extensions for semilinear elliptic equations in $\mathbb{R}^n$

Marco Cappiello

Dipartimento di Matematica,
Università di Torino
Via Carlo Alberto 10, 10123, Torino
Italy

marco.cappiello@unito.it

Abstract: We derive a description of regularity and decay properties of solutions of a large class of semilinear elliptic pseudodifferential equations in $\mathbb{R}^n$ with coefficients admitting irregular decay at infinity. Representative models are Schrödinger equations of the form

$$-\Delta u + \frac{\omega(x)}{\langle x \rangle^\sigma} u = f + F[u], \qquad x \in \mathbb{R}^n,$$

where $0 < \sigma < 2$, $\langle x \rangle = (1 + |x|^2)^{1/2}$, $\omega(x)$ a bounded smooth function, f given and $F[u]$ a polynomial in u, and similar Schrödinger equations at the endpoint of the spectrum. Other relevant examples are given by linear and nonlinear ordinary differential equations with irregular type of singularity for $x \to \infty$, admitting solutions $y(x)$ with holomorphic extension in a strip and sub-exponential decay of type $|y(x)| \leq Ce^{-\varepsilon|x|^r}$, $0 < r < 1$. Decay and uniform holomorphic extensions are obtained in terms of Gelfand-Shilov spaces by an inductive technique. The results presented have been obtained in collaboration with T. Gramchev and L. Rodino, see [1].

References

[1] M. Cappiello, T. Gramchev and L. Rodino, *Sub-exponential decay and uniform holomorphic extensions for semilinear pseudodifferential equations*, Comm. Partial Differential Equations **35** (2010) n. 5, 846–877.