A class of Fourier integral operators with complex phase related to the Gevrey classes

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Abstract:

We discuss about Fourier integral operators with complex phase functions belonging to $S^{\kappa}_{\rho,\delta}$, $0 < \delta < \rho \le 1$, $0 < \kappa < \rho - \delta$ where the positivity/negativity of the phase functions is not assumed. In particular we prove composition formulae for 0 and 1 quantization of Fourier integral operators with phase function ϕ and $-\phi$ where ϕ verifies the estimates;

$$|\partial_x^{\beta}\partial_{\xi}^{\alpha}\phi(x,\xi)| \le CA^{|\alpha+\beta|}|\alpha+\beta|!^{s}\langle\xi\rangle^{\kappa+\delta|\beta|-\rho|\alpha|}$$

with $s > 1$ close to 1.

We show how this composition formula is applied to obatain energy estimates for a second order noneffectively hyperbolic operator;

$$P = -D_0^2 + \phi_1(x,D)D_0 + \phi_2^2(x)\langle D\rangle^2$$

where $\phi_1(x,\xi)$ and $\phi_2(x)$ are assumed to verify $\{\phi_1, \phi_2\} \ge c > 0$ and prove that the Cauchy problem for P is Gevrey s well-posed for any lower order term provided $1 \le s < 4$. Ineed the conjugation of Fourier integral operators with the phase function

$$\phi(x,\xi) = \langle\xi\rangle^{\kappa}\log\left(\phi_2(x) + \sqrt{\phi_2(x)^2 + \langle\xi\rangle^{-1}}\right) \in S^{1/4}_{1,1/2}$$

with $\kappa < 1/4$ transforms P to another operator for which we can easily get apriori estimates in Sobolev spaces .