Wave Operators for the Matrix Zakharov-Shabat System

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Abstract

Consider the matrix Zakharov-Shabat system

$$iJ\frac{\partial X}{\partial x}(x,\lambda) - V(x)X(x,\lambda) = \lambda X(x,\lambda),$$

where $x \in \mathbb{R}$ is position, λ is the spectral variable, $J = I_m \oplus (-I_n)$, and the potential

$$V(x) = \begin{pmatrix} 0_{m\times m} & iq(x) \\ ir(x) & 0_{n\times n} \end{pmatrix}$$

is the (complex) potential having its entries in $L^1(\mathbb{R})$. This equation occurs in signal propagation in optical fibers and in wave propagation on the surface of deep waters when $r(x) = \pm q(x)^*$. For this equation we develop a time-dependent scattering theory analogous to the theory existing for the Schrödinger and Dirac equations. We prove the existence and asymptotic completeness of the wave operators without "eigenvalues imbedded in the continuous spectrum." The scattering operator is unitarily equivalent, by Fourier transformation, to the scattering matrix occurring in the prevalent scattering theory of the matrix Zakharov-Shabat system. Because the differential system is in general nonselfadjoint, we cannot rely on the usual time-dependent scattering theory but instead have to rely on Kato's theory of relative smoothness. As a net result, we prove that the matrix Zakharov-Shabat Hamiltonian $iJ(d/dx) - V$ is the direct sum of a matrix having no real eigenvalues and an unbounded operator similar to the free Hamiltonian $iJ(d/dx)$.

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