

Lefschetz fibration structures on 4-dimensional 2-handlebodies

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Abstract:

Lefschetz fibrations on 4-manifolds are smooth maps $f : M^4 \rightarrow S$, where S is a surface, with finitely many singular points where the local model is $f(z, w) = z^2 + w^2$ or $f(z, w) = z^2 + \bar{w}^2$ with respect to suitable local complex coordinates. In the bounded case we require also that $f|_1 : M^4 - f^{-1}(\Sigma_f) \rightarrow S - \Sigma_f$ is a fibre bundle, where Σ_f is the set of singular values of f .

We assume M^4 to be a compact oriented 4-manifold with boundary obtained from the 4-ball by the addition of 1- and 2-handles (the so called 4-dimensional 2-handlebodies). By a theorem of J. Harer, Lefschetz fibrations over the disc exist on such 4-manifolds.

We give a complete set of moves which relate any two Lefschetz fibrations $f_1, f_2 : M^4 \rightarrow B^2$, where B^2 is the disc. The only assumption on f_1 and f_2 is to induce 2-handlebodies which can be related by classical Kirby moves. These are a special class of diffeomorphisms, and it is a subtle open question whether Kirby moves coincide with all the diffeomorphisms.

Applications to open books on 3-manifolds are given.