Sub-Exponential Decay and Holomorphic Extensions for Elliptic Pseudodifferential Equations on $\mathbb{R}^n$

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Abstract: The goal of the present talk is to derive a simultaneous description of the decay and the regularity properties for elliptic equations in $\mathbb{R}^n$ with coefficients admitting irregular decay at infinity of the type $O(|x|^\sigma), \sigma > 0$, filling the gap between the case of Cordes globally elliptic operators and the case of regular/Fuchs behavior at infinity. Representative examples in $\mathbb{R}^n$ are the equations

$$-\triangle u + \frac{\omega(x)}{\langle x\rangle^\sigma} u = f + F[u], x \in \mathbb{R}^n$$

where $0 < \sigma < 2$, $\langle x\rangle = (1+|x|^2)^{1/2}$, $\omega(x)$ a bounded smooth function, f given and $F[u]$ a polynomial in u, and similar Schrödinger equations at the endpoint of the spectrum. Other relevant examples are given by linear and nonlinear ordinary differential equations with irregular type of singularity for $x \to \infty$, admitting solutions $y(x)$ with holomorphic extension in a strip and sub-exponential decay of type $|y(x)| \leq Ce^{-\varepsilon|x|^r}; 0 < r < 1$. Sobolev estimates for the linear case are proved in the frame of a suitable pseudodifferential calculus; decay and uniform holomorphic extensions are then obtained in terms of Gelfand-Shilov spaces by an inductive technique. The same technique allows to extend the results to the semilinear case.

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