THE CAUCHY PROBLEM FOR SCHRÖDINGER-TYPE EQUATIONS WITH DATA IN GELFAND-SHILOV SPACE

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Abstract

We consider the initial value problem

$$\begin{cases} P(t,x,\partial_t,\partial_x)u(t,x) = f(t,x) \\ u(0,x) = u_0(x) \end{cases}, \quad (t,x) \in [0,T] \times \mathbb{R}^n \tag{1}$$

where

$$P(t,x,\partial_t,\partial_x) = \partial_t - i\triangle_x + \sum_{j=1}^{n} a_j(t,x)\partial_{x_j} + b(t,x). \tag{2}$$

It is well-known that when the coefficients a_j, b and the Cauchy data f, u_0 are all real valued, smooth and uniformly bounded with respect to x the Cauchy problem (1) is L^2-well-posed, while if a_j are complex valued suitable decay conditions for $|x| \to \infty$ are needed on the imaginary part of the coefficients in order to obtain either H^∞ or Gevrey well posedness with a certain loss of derivatives. Here we study how an exponential behavior of the initial data influences the regularity and behavior of the solution for $|x| \to \infty$. In particular, we consider the case when the initial data belong to the Gelfand-Shilov space $\mathcal{S}_s^\theta(\mathbb{R}^n)$, (resp. $\Sigma_\theta^s(\mathbb{R}^n)$) defined as the space of the smooth functions f satisfying

$$\sup_{x\in\mathbb{R}^n} \sup_{\alpha\in\mathbb{N}^n} C^{-|\alpha|}\alpha!^{-\theta} e^{c|x|^{\frac{1}{s}}} |\partial^\alpha f(x)| < \infty,$$

for some (resp. for all) $C, c > 0$, with $s > 1, \theta > 1$, and prove results of existence and uniqueness of the solution of (1) with precise information both on the regularity and on the behavior of the solution for $|x| \to \infty$. Finally we outline some possible generalizations of these results to a more general class of operators including (2).

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