FROM BERNOULLI PARTIAL SHIFT TO THE PARTIAL ALGEBRA $\mathbb{K}_{par}$

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Resumo

Let G a group (finite or not). Taking the sets of its finite subsets - denoted by $P(G)$-, one can easily induce an action $\mathfrak{B} : G \curvearrowright P(G)$ by $\mathfrak{B}_g(A) = gA$, where $A \in P(G)$ and $gA = \{ga; a \in A\}$. Such action is called *Bernoulli global shift*; the "global" included will be explained. Although basic, this action hides a nice feature: one can restrain this action to a partial one, by using only the finite subsets of G which contain the neutral element $e \in G$; we denote this subset of $P(G)$ by $P_e(G)$.

It's an appropriate time to a (short) historical background and some theory development. Group actions are a common tool and are used by many mathematical fields. Despite this fact, like the group defining operation, these actions must be global defined. What does this mean? The answer is: the domain of group actions is the whole group.

As we emphasized the term "global", now we introduce the idea of (the above commented) restricted actions; this leads us to "partial actions" - and inverse semigroups, and groupoids. The concept of a partial action was coined by Dr. Ruy Exel in his area of expertise - C^* algebras -, in the decade of 1990. A few years later, in collaborative work with Dr. Mikhailo Dokuchaev, they created a (purely) algebraic version. Since the publication of such work, many mathematicians have been studying and developing new uses of partial actions.

Once again we state: if one restricts $\mathfrak{B} : G \curvearrowright P(G)$ to subsets contained $e \in G$, what we get is the *Bernoulli partial shift* $\mathfrak{b} : G \curvearrowright P_e(G)$ by $\mathfrak{b}_g : D_{g^{-1}} \to D_g$ with $\mathfrak{b}_g(A) = gA$, where $D_h := \{A \in P(G); A \ni e, h\}$. (In fact: any partial group action is the restriction of a global gorup action.)

Once we know what a partial action means, we move to the "partial algebra". In a further collaboration with Dokuchaev and Dr. Paollo Piccone, Exel realized: the same way representations of a finite group G on a $\mathbb{K}$-vector spaces can be made using the group algebra $\mathbb{K}G$, there is a version of partial representations and a respective *partial group algebra* - which they denoted by $\mathbb{K}_{par}(G)$. Moreover, their work has a formula to compute partial algebras.

Though the Bernoulli partial shift and the partial algebra may be seen not related, we devote this talk to present its interaction. To accomplish this task we are going to talk about two structures one can define from $\mathfrak{b} : G \curvearrowright P_e(G)$: an inverse semigroup S_{PB}, and a groupoid Γ_{PB} - where PB stands for "P"artial "B"ernoulli. Both are different manners to generalize groups.

More specifically we are going to show that the partial algebra is isomorphic to the inverse semigroup algebra, which is isomorphic to the groupoid algebra, ie. $\mathbb{K}_{par}(G) \simeq \mathbb{K}S_{par} \simeq \mathbb{K}\Gamma_{PB}$. This provides an alternative way to achieve the formula of Exel et al, via inverse semigroup theory (using Green's relations).

This is an unpublished work; a chapter of the speaker's Ph.D. thesis under the advisiorship of Dr. Marcelo Muniz Alves focused on: Algebras os expanded structures.

References

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