RATE OF CONVERGENCE OF ATTRACTORS OF REACTION-DIFFUSION EQUATIONS WITH NONLINEAR BOUNDARY CONDITIONS

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Abstract

This paper we study the rate of convergence of the asymptotic dynamics of reaction-diffusion equations with nonlinear Robin boundary conditions. We show how the rate of convergence of the global attractors can be affected by the variation of the potentials, boundary conditions and vectors fields. In fact we consider the family of parabolic equations

(1)
$$\begin{cases} \partial_t u^{\varepsilon} - \operatorname{div}(p_{\varepsilon}(x)\nabla u^{\varepsilon}) + (\lambda + V_{\varepsilon}(x))u^{\varepsilon} = f^{\varepsilon}(u^{\varepsilon}) & \text{in } \Omega \times (0,\infty), \\ \frac{\partial u^{\varepsilon}}{\partial \vec{n}_{\varepsilon}} + (\lambda + b_{\varepsilon}(x))u^{\varepsilon} = g^{\varepsilon}(u^{\varepsilon}) & \text{on } \Gamma \times [0,\infty), \end{cases}$$

under standard conditions and the following convergence conditions

$$0 < m_0 \leqslant p_{\varepsilon}(x) \quad \text{and} \quad \|p_{\varepsilon} - p_0\|_{L^{\infty}(\Omega)} \to 0 \text{ as } \varepsilon \to 0^+,$$

$$\|V_{\varepsilon} - V_0\|_{L^{\infty}(\Omega)} \leqslant \eta(\varepsilon) \quad \text{and} \quad \|b_{\varepsilon} - b_0\|_{L^{\infty}(\Gamma)} \leqslant \tau(\varepsilon),$$

$$\|f^{\varepsilon}(u) - f^0(u)\|_{L^2(\Omega)} \leqslant \kappa(\varepsilon) \quad \text{and} \quad \|g^{\varepsilon}(v) - g^0(v)\|_{H^{-\frac{1}{2}}(\Gamma)} \leqslant \xi(\varepsilon),$$

for appropriated functions $p_{\varepsilon}, \eta, \kappa, \tau$ and ξ . We will prove that the convergence of eigenvalue, eigenfunction, equilibrium points, spectral projections, local invariant unstable manifolds and global attractors as $\varepsilon \to 0^+$ of the problem (1) can be estimate by

$$[\|p_{\varepsilon} - p_0\|_{L^{\infty}(\Omega)} + \eta(\varepsilon) + \tau(\varepsilon) + \kappa(\varepsilon) + \xi(\varepsilon)]^l, \quad N \ge 2, \ 0 < l < 1$$

and for the scalar situation N = 1 we can estimate by

$$|\log(||p_{\varepsilon} - p_0||_{L^{\infty}(\Omega)} + \eta(\varepsilon) + \tau(\varepsilon) + \kappa(\varepsilon) + \xi(\varepsilon))|[||p_{\varepsilon} - p_0||_{L^{\infty}(\Omega)} + \eta(\varepsilon) + \tau(\varepsilon) + \kappa(\varepsilon) + \xi(\varepsilon)].$$

This is a joint work with Prof.Dr. Flank David Morais Bezerra - UFPB.

References

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